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A Discussion of the Importance of Line Tension on Cottrell's Theory of the Sharp Yield Point

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The activation energy required to break a pinned dislocation line away from its condensed atmosphere of impurity atoms is calculated as a function of applied stress, without neglecting line tension. Reasons are presented for not assuming the effect of line tension to be negligible when considering the problem of freeing a small segment of dislocation from an impurity atmosphere. The dislocation is considered to be pinned at each atomic site along its length and the pinning points are assumed immobile. The development is a refinement of the original Cottrell-Bilby[1] theory. The solution is compared to recent approximations developed by Cottrell[3] and Haasen[5] in which line tension considerations were neglected. A qualitative explanation of the yielding process in terms of the activation of Frank-Read sources is presented, but the lack of a realistic solution to the dynamic dislocation problem involved prevents an extension of the model at present. A self-consistent correlation between the present calculations and experimental data for delay time associated with yielding and the temperature dependence of the upper yield stress was made. Favorable agreement was noted. It is concluded that extension of the original Cottrell-Bilby[1] theory which includes line tension effects can just as well describe the yielding process as other approximations[3,5] which neglect line tension.

In 1949 Cottrell and Bilby[1] considered the problem of determining the force required to pull a dislocation away from its condensed atmosphere of impurity atoms. They also examined the effect of thermal fluctuations on this force. The dislocation line was pictured as bowing out in a triangular loop under the action of an applied stress, and the activation energy necessary to free the dislocation from its atmosphere was calculated as a function of stress, line energy, and dislocation-solute atom interaction.

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Cottrell and Bilby found the form of the activation energy vs stress curves to be almost independent of the line energy and the interaction energy (*i.e.*, these parameters only affected the activation energy U by a scale factor). A check of the theory was accomplished by comparison with the experimental data of McAdam and Mebs[2] for the lower yield stress as a function of temperature.

Cottrell,[3] in 1957, attempted to obtain a simple closed-form solution for the activation energy as a function of stress by linearizing the quartic equation relating displacement of the dislocation line and applied stress. It was argued that since the contribution to the line energy/atomic plane of a dislocation is made by the long-range stress field of the dislocation through a term of the type $(Gb^3/2\pi)\ln \Lambda/r_0$, where

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Λ = width of the dislocation loop
r_0 = **b**, the Burgers vector
G = the shear modulus

and since Λ is small considering the loop sizes involved in the yielding process, then this line energy contribution is small enough to be neglected.

Comparison of the theory with the data of Clark and Wood[4] for delay times associated with yielding as a function of stress was made, and fair agreement was noted except at high temperatures, where presumably strain aging effects were present.

Haasen[5] refined Cottrell's linearized theory by solving the quartic equation relating stress and dislocation displacement for low values of stress and deduced a T^{-3} temperature dependence of the breakaway stress in place of Cottrell's $T^{1/3}$ dependence. This calculation was also made on the assumption that line tension effects are negligible, and Haasen pointed out that there was very little difference between the (σ, T) relationship deduced by himself and that obtained by Cottrell.

In the same paper Haasen considered the effect of line tension on the athermal yield stress after aging in tension and in compression and was theoretically able to explain data obtained for the Fe-C and Zn-N system. Haasen concluded that while line tension is important in athermal yielding, it has a negligible effect with regard to the thermal freeing of a continuously pinned dislocation line.[5]

The present authors undertook a rigorous numerical solution to the problem formulated by Cottrell and Bilby[1] as a means of checking the approximate solutions given by Cottrell[3] and Haasen.[5] Line tension was not neglected in these calculations. Cottrell[3] argues that the contribution of the line energy through the term $(G\mathbf{b}^3/2\pi)\ln \Lambda/r_0$ is small since the loop lengths associated with yielding are only several atomic spacings in length. But even if Λ is considered small (later calculations showed that $\Lambda \simeq$ 6-8**b**), $\ln \Lambda/r_0 \simeq$ 1-2, a finite value.

One must also consider the contribution of the core energy of the dislocation to the line tension term. Thus, regardless of the small size of the dislocation loops considered, the line energy contribution will be finite since the core energy is independent of Λ, the wavelength of the undulation of the dislocation line. The sum of the elastic strain energy and the core energy for small loops is thus about 0.1 to 0.2 of the values generally employed in considering line tension[9] (*i.e.*, about $G\mathbf{b}^3$ per atomic plane where G is shear modulus and **b** the Burgers vector).

The scope of the present paper is to investigate in detail certain consequences of the theory of the sharp yield point as developed through the work of Cottrell and Bilby,[1] Cottrell,[3] and Haasen[5] when line tension is considered in detail. It is not intended that this paper should in any way try to establish the validity of this theory over other theories due to Louat[6,7] or Lothe[8] concerning the sharp yield point. The later theories use dislocation models considerably different from that suggested by Cottrell and Bilby[1] and it would go beyond the scope of this paper to discuss the merits and limitations of each model. The present authors do believe, however, that the original Cottrell-Bilby[1] theory contains a reasonably adequate physical model for the yield phenomena.

REFINED TREATMENT OF THE BREAKAWAY PROBLEM

Cottrell and Bilby[1] originally discussed the problem of pulling a dislocation line away from its condensed atmosphere by considering that the activation energy necessary for freeing the dislocation is dependent upon three energy terms:

i) The interaction of the dislocation with its atmosphere.
ii) The increase in line energy due to loop formation.
iii) The work done by the dislocation as it bows out under the applied stress.

If ψ defines the acute angle between the triangular dislocation loop* and the straight dislocation line, and the activation energy expression is minimized with respect to ψ, Cottrell and Bilby showed that:

*The triangular-shaped loop is considered a good approximation to a circular or elliptical shaped arc. At present the most accurate configuration of the dislocation loop is not known.

$$\psi = \cos^{-1}\frac{\beta}{\alpha} \qquad [1]$$

$$U = \{\alpha^2 - \beta^2\}^{1/2} \qquad [2]$$

where

$$\alpha = 2W(x_2 - x_1) \qquad [3]$$

$$\beta = \frac{2A}{\rho}\left\{\tan^{-1}\frac{x_2}{\rho} - \tan^{-1}\frac{x_1}{\rho}\right\} + 2\{U_i(x_1) + W\} \times (x_2 - x_1) + F(x_2 - x_1)^2 \qquad [4]$$

U = the activation energy in ergs

W = the line energy in ergs per cm

x_1 = the displacement in cm (in the slip direction) of the dislocation at an applied stress σ, relative to its position at $\sigma = 0$.

x_2 = the displacement in cm (at an applied stress σ) of a dislocation loop large enough to cause catastrophic breakaway of the entire dislocation line from its atmosphere.

$U_i(x_1)$ = the atmosphere-dislocation interaction energy in ergs per cm at displacement x_1.

ρ = the distance ($\simeq$ 2Å) between the dislocation line and a solute atom of the condensed atmosphere located below the dislocation at $\theta = 3\pi/2$.

F = the external force in dynes per cm on the dislocation line due to an applied stress σ.

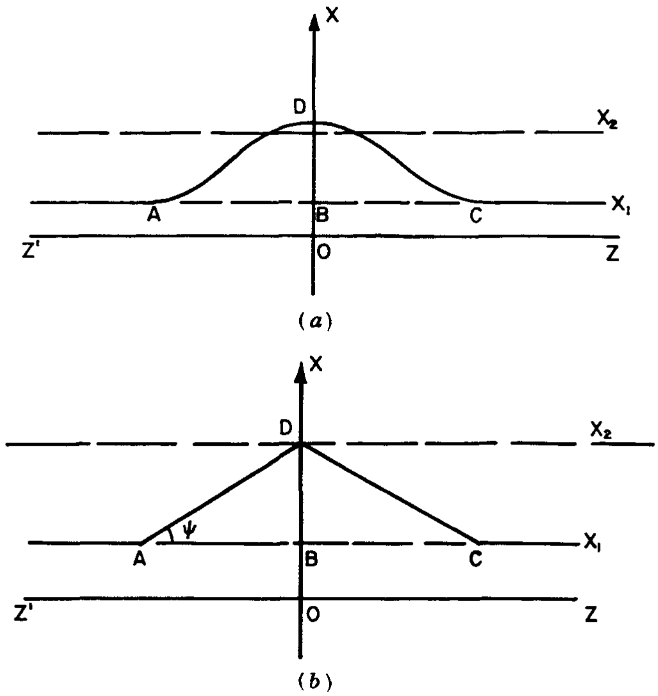

Fig. 1—Geometry of loop nucleation problem after Cottrell and Bilby.[1]

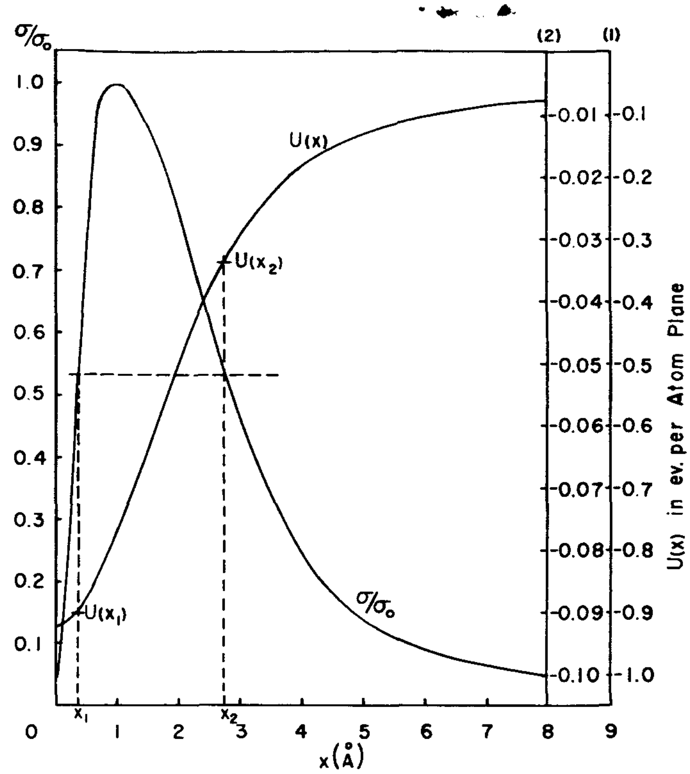

Fig. 2—Variation of interaction energy (U) and applied stress (σ/σ_0) with dislocation position after Cottrell and Bilby.[1] Scale (1) $A = 3 \times 10^{-20}$ erg cm; $\rho = 2$Å. Scale (2) $A = 3 \times 10^{-21}$ erg cm; $\rho = 2$Å.

A is defined by the dislocation-solute interaction strength and is a function of the elastic constants of the material and the type of impurity present

A is assumed to be about 3×10^{-20} or 3×10^{-21} erg-cm. Fig. 1 taken from the paper by Cottrell and Bilby[1] clearly shows the geometry of the problem. In this figure a positive edge dislocation is considered to lie along the Z axis and the Burgers vector of the dislocation lies in the X axis. When the applied stress is zero the equilibrium position of the dislocation is ZZ′, vertically above the line of the atmosphere. Under an applied stress σ the dislocation moves forward to the line x_1. The bulge ADC, Fig. 1(a), represents the dislocation loop which must be nucleated for breakaway. The latter is approximated by the triangular loop ACD shown in Fig. 1(b). Fig. 2, also taken from the paper by Cottrell and Bilby,[1] shows the variation of interaction energy (U_i) and applied stress (σ/σ_0) with dislocation position. Cottrell and Bilby[1] claimed that the activation energy could be shown to be of the form

$$U = D(\sigma/\sigma_0)\{A\,E(\sigma/\sigma_0)[2W\rho - A\,E(\sigma/\sigma_0)]\}^{1/2} \qquad [5]$$

where D and E do not depend on the physical parameters A, W, and ρ. The present authors were unable readily to verify this relation and thus proceeded rigorously to solve the problem numerically.

The force on the dislocation line, $F_x = -\partial U_i(x)/\partial x$ exhibits a maximum at $x = x_0 = \rho/\sqrt{3}$. Calling σ_0 the critical tensile stress at $x = x_0$, it is possible to express the activation energy as

$$U = \frac{2A}{\rho}\{\alpha'^2 - \beta'^2\}^{1/2} \qquad [6]$$

$$\alpha' = \frac{W\rho^2}{A}\left(\frac{\lambda}{\sqrt{3}}\right) \qquad [7]$$

$$\beta' = \tan^{-1}\frac{\lambda/\sqrt{3}}{1+\frac{\theta}{3}} + \frac{\lambda}{\sqrt{3}}\left\{\frac{W\rho^2}{A} - \frac{1}{1+\frac{1}{12}\left(\frac{\sigma}{\sigma_0}\right)^2}\right\} + \frac{3\sqrt{3}}{16}\left(\frac{\sigma}{\sigma_0}\right)\left(\frac{\lambda}{\sqrt{3}}\right)^2 \qquad [8]$$

$$\lambda = \frac{x_2}{x_0} - \frac{x_1}{x_0} \qquad [9]$$

$$\theta = \left(\frac{x_2}{x_0}\right)\left(\frac{x_1}{x_0}\right) \qquad [10]$$

The only approximation introduced up to this point was Haasen's approximation for x_1 in the term $U_i(x_1)$. The error introduced is negligible for $\sigma/\sigma_0 \leq 0.60$, and is only 4 pct at $\sigma/\sigma_0 = 0.85$.

The activation energy was determined as a function of σ/σ_0 by finding x_2/x_0 and x_1/x_0 for corresponding values of σ/σ_0 from an accurate graph of

$$\frac{\sigma}{\sigma_0} = \frac{16}{9}\frac{x/x_0}{\left\{1+\frac{1}{3}\left(\frac{x}{x_0}\right)^2\right\}^2} \qquad [11]$$

This last equation is the quartic which Haasen solved for $\sigma/\sigma_0 \ll 1$. For σ/σ_0 near 0.50, however, a check of Haasen's approximation revealed errors in x_2 as large as 30 pct.

The resulting activation energy vs stress curves

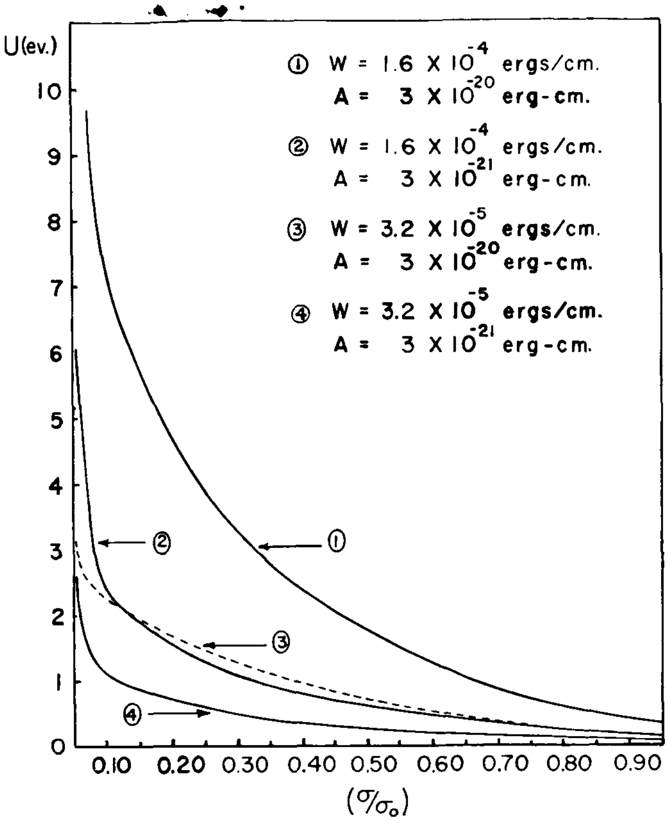

Fig. 3—Activation energy vs σ/σ_0 for various values of the parameters A and W.

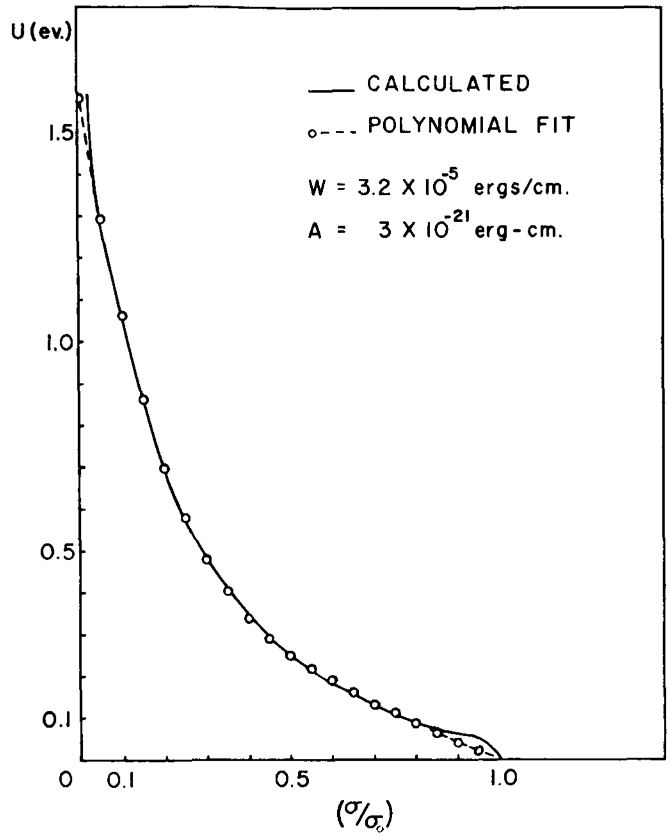

Fig. 4—Polynomial fit to $U(\sigma/\sigma_0)$. (Case 4)

for four combinations of the parameters A and W are shown in Fig. 3. These curves closely resemble the $U(\sigma/\sigma_0)$ plots shown in Fig. 4 of the Cottrell-Bilby article, except at low stress levels where the activation energies appear to differ by nearly 1 ev.

An attempt to obtain an analytical expression for the stress dependence of the activation energy was accomplished by fitting a fourth order polynomial in σ/σ_0 to each of the activation energy curves. Fig. 4 shows a typical fit of a polynomial to the derived curve for $A = 3\times10^{-21}$ erg-cm and $W = 3.2\times10^{-5}$ erg per cm. It is noted that the boundary condition $U(\sigma/\sigma_0 = 1) = 0$ is satisfied, but $U(\sigma/\sigma_0 = 0)$ approaches a finite value instead of becoming infinitely large. If we restrict the region of interest to $0.10 \le \sigma/\sigma_0 \le 0.85$, the polynomials are excellent representations to the derived curves. The following set of fourth order polynomials were found to fit for the specified values of A and W.

Case 1. $A = 3\times10^{-20}$ erg-cm; $W = 1.6\times10^{-4}$ erg/cm

$$U(\text{ev}) = 10.77 - 42.74\left(\frac{\sigma}{\sigma_0}\right) + 78.79\left(\frac{\sigma}{\sigma_0}\right)^2 - 70.54\left(\frac{\sigma}{\sigma_0}\right)^3 + 23.72\left(\frac{\sigma}{\sigma_0}\right)^4 \quad [12]$$

Case 2. $A = 3\times10^{-21}$ erg-cm; $W = 1.6\times10^{-4}$ erg/cm

$$U(\text{ev}) = 3.615 - 14.57\left(\frac{\sigma}{\sigma_0}\right) + 26.70\left(\frac{\sigma}{\sigma_0}\right)^2 - 22.67\left(\frac{\sigma}{\sigma_0}\right)^3 + 6.93\left(\frac{\sigma}{\sigma_0}\right)^4 \quad [13]$$

Case 3. $A = 3\times10^{-20}$ erg-cm; $W = 3.2\times10^{-5}$ erg/cm

$$U(\text{ev}) = 3.038 - 9.036\left(\frac{\sigma}{\sigma_0}\right) + 13.62\left(\frac{\sigma}{\sigma_0}\right)^2 - 11.20\left(\frac{\sigma}{\sigma_0}\right)^3 + 35.75\left(\frac{\sigma}{\sigma_0}\right)^4 \quad [14]$$

Case 4. $A = 3\times10^{-21}$ erg-cm; $W = 3.2\times10^{-5}$ erg/cm

$$U(\text{ev}) = 1.591 - 6.449\left(\frac{\sigma}{\sigma_0}\right) + 12.09\left(\frac{\sigma}{\sigma_0}\right)^2 - 10.98\left(\frac{\sigma}{\sigma_0}\right)^3 + 3.75\left(\frac{\sigma}{\sigma_0}\right)^4 \quad [15]$$

A calculation was made to determine the length (l) of the dislocation loop which must be thermally activated as a function of stress. This is done by numerically evaluating one of the following relations as a function of stress

$$l = 2(x_2 - x_1)\cot\psi \quad [16]$$

or

$$\frac{l}{x_0} = 2\lambda\cot\psi \quad [17]$$

It was found that for:

Case 1. $W = 1.6\times10^{-4}$ erg/cm; $A = 3\times10^{-20}$ erg-cm l was approximately constant ($\simeq$ 11.5Å) for $0.15 \le \sigma/\sigma_0 \le 0.65$ and decreased to 5Å at $\sigma/\sigma_0 = 0.95$.

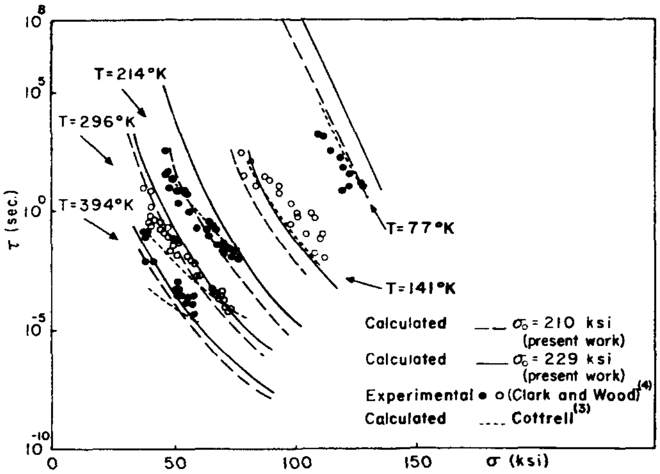

Fig. 5—Delay time vs stress for various temperatures. (Experimental data after Clark and Wood).[4]

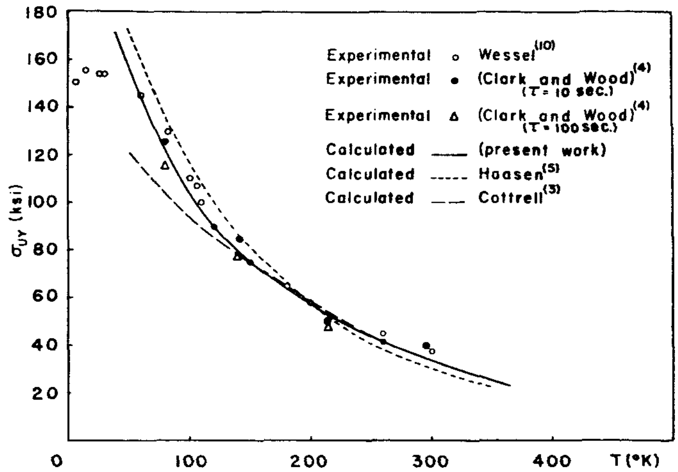

Fig. 6—Upper yield stress vs temperature. (Experimental data after Wessel,[10] and Clark and Wood).[4]

Case 2. $W = 1.6 \times 10^{-4}$ erg/cm; $A = 3 \times 10^{-21}$ erg-cm l decreased with increasing stress; $l \simeq 46\text{Å}$ at $\sigma/\sigma_0 = 0.10$ and decreased to 10Å at $\sigma/\sigma_0 = 0.95$.

Case 3. $W = 3.2 \times 10^{-5}$ erg/cm; $A = 3 \times 10^{-20}$ erg-cm l increased from 1Å at $\sigma/\sigma_0 = 0.20$ to a maximum of 4Å at $\sigma/\sigma_0 = 0.65$ and then decreased to 1.5Å at $\sigma/\sigma_0 = 0.95$.

Case 4. $W = 3.2 \times 10^{-5}$ erg/cm; $A = 3 \times 10^{-21}$ erg-cm l was approximately constant (16Å) for $0.20 \leq \sigma/\sigma_0 \leq 0.65$ and decreased to 6Å at $\sigma/\sigma_0 = 0.95$.

The peculiar behavior of l as a function of σ/σ_0 for Case 3 raises the interesting question as to the range of values for A and W where this model is valid. Since U must be positive, α must be $\geq |\beta|$, Eq. [2], or α' must be $\geq |\beta'|$, Eq. [6]. The authors have examined the two limiting cases, λ approaching 0 and consequently σ/σ_0 approaching 1, and λ approaching ∞ and consequently σ/σ_0 approaching 0. It was found that:

$$\lim_{\lambda \to 0} \left|\frac{\beta'}{\alpha'}\right| = 1 - \frac{9}{52}\frac{A}{W\rho^2}$$

and

$$\lim_{\lambda \to \infty} \left|\frac{\beta'}{\alpha'}\right| = 1 - \frac{A}{W\rho^2}$$

Therefore for $|\beta'/\alpha'|$ to be ≤ 1, and thus not violate the conditions for formulation of the problem, $W\rho^2/A \geq 0.5$. When the latter condition is satisfied, α' is always $\geq |\beta'|$ and the analysis is valid. This condition was violated in Case 3 where $W\rho^2/A = 0.427$. Still $\alpha' \geq |\beta'|$ except for $\sigma/\sigma_0 < 0.10$. For this reason, however, Case 3 is not considered generally valid and is not used for fitting experimental data.

CORRELATION WITH EXPERIMENTAL DATA

a) Delay Time Data. An attempt was made to fit the theory with the data of Clark and Wood[4] and to compare the fit with that of Cottrell.[3] Using the conventional Arrhenius relation

$$\tau = \tau_0 \exp\{U(\sigma)/kT\} \quad [18]$$

or

$$\tau = \frac{\bar{l}}{\nu b} \exp\{U(\sigma)/kT\} \quad [19]$$

where

τ = the delay time for yielding
ν = the Debye frequency $\simeq 10^{13}$ sec^{-1}
$\bar{l}$ = the average loop length $\simeq$ 16Å (Case 4)
k = Boltzmann's constant, and
T = the absolute temperature in °K;

curves for τ vs σ at fixed T were obtained and are given in Fig. 5. It is noted that the expression $\tau_0 = \bar{l}/\nu b$ arises from Friedel's[9] formula for the period of free oscillations of a dislocation line. τ_0 was calculated to be 8×10^{-13} sec when Case 4 is considered.

The calculated delay time-stress curves showed better agreement with the data at high temperatures than did Cottrell's linear theory, while both theories were similar fits at low temperatures. For a value of $A = 3 \times 10^{-21}$ erg-cm, σ_0 is readily calculated as 229,000 psi. Cottrell arbitrarily chose values of 310,000 psi and 10^{-11} sec for σ_0 and τ_0 respectively, in showing agreement between theory and experimental delay time-stress curves.

b) Temperature Dependence of the Upper Yield Stress. A further check of this refined calculation was accomplished by comparison with the published data of Wessel[10] and Erickson and Low[4] concerning the temperature dependence of the upper yield stress. For a constant delay time:

$$\frac{U}{kT} = \ln\left\{\frac{\tau}{\tau_0}\right\} = \text{constant} \quad [20]$$

Taking the expression for $U(\sigma)$ from the present work (Case 4), and the corresponding expressions due to Cottrell[3] and Haasen[5] and using σ_0 = 229,000

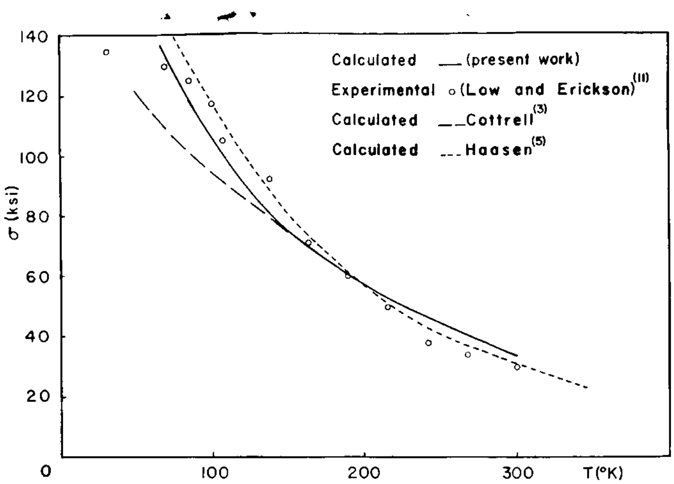

Fig. 7—Upper yield stress vs temperature. (Experimental data after Erickson and Low).[11]

psi, it is possible to plot calculated curves of σ_{UY} vs temperature for each of the three types of analysis. These are shown in Figs. 6 and 7. The values of $\ln\{\tau/\tau_0\}$ for the present work, (Cottrell's[3] and Haasen's[5]), used in these figures were 33.5, 21.7, and 10, respectively. These values were selected to force the calculated curves to fall upon the experimental curves at 200°K. Also shown in Fig. 6 is the temperature dependence of the flow stress at two constant delay times from the work of Clark and Wood.[4] There is very little difference between Haasen's approximation and that of the present work with respect to success in fits to the experimental curves. Cottrell's approximation does seem to predict too shallow a temperature dependence at relatively low temperatures.

DISCUSSION AND SUMMARY

The dislocation strain rate at the upper yield stress is considered to be given as:

$$\dot{\gamma} = \rho_s n_s \mathbf{b}\bar{x}\,\frac{\nu\mathbf{b}}{\bar{l}}\exp\left\{-\frac{U(\sigma)}{kT}\right\} \qquad [21]$$

where ρ_s is the density of Frank-Read sources in cm^{-2}, n_s the number of dislocation loops produced per source during yielding, $\bar{x}$ the average distance moved by the generated dislocation in the incipient Lüders band after catastrophic breakaway, and the other terms have been previously described. The dislocation model envisioned is one of catastrophic breakaway of a small dislocation loop of length l to a length L_N, the distance between nodal points of the dislocation network in an incipient Lüders band. These lengths of dislocation lines are now effective Frank-Read sources. The delay time τ is interpreted as the time required for a statistically large number of these sources to become activated. It is then assumed that each activated source is capable of generating many more loops (n_s) almost instantaneously.

It is clear that the values of the constant $\ln\{\tau/\tau_0\}$ found for the various approximations discussed in this paper may help to decide which approximation is most realistic. From Eq. [21] the constant under consideration is then given by

$$\ln\left\{\frac{\tau}{\tau_0}\right\} = \ln\left\{\frac{\rho_s n_s \mathbf{b}\bar{x}}{\dot{\gamma}\tau_0}\right\} \qquad [22]$$

The experimental data for the upper yield point of Wessel[10] and Erickson and Low[11] were taken at a macroscopic strain rate of 2.1×10^{-3} and 6.67×10^{-4} sec^{-1} respectively. It seems reasonable to assume that the corresponding dislocation strain rate at an incipient Lüders band is about 2×10^{-1} sec^{-1}. This is because the appropriate gage length at this point of the stress-strain diagram would be the width of the incipient Lüders band (about 0.01 in.), not the normally used gage length of about 1.0 in. τ_0 was previously evaluated at 8×10^{-13} sec. If one considers $\rho_s \simeq 10^8$ cm^{-2}, $n_s \simeq 10^3$ loops/source and $\bar{x} \simeq 10^{-2}$ cm, $\ln\{\tau/\tau_0\}$ is $\simeq 32$. This value compares favorably with the value 33.5 chosen to fit the present calculations with the experimental data. Even if one considers $\rho_s = 10^6$ cm^{-2}, $n_s = 1$, $\bar{x} = 1\times10^{-4}$ cm, $\dot{\gamma} = 10^{-1}$ sec^{-1}, and $\tau_0 = 8\times10^{-13}$ sec, then $\ln\{\tau/\tau_0\}$ becomes equal to 14.6, which is considered a minimum reasonable value. Haasen's approximation does not seem to be suitable because it predicts too low a value of $\ln\{\tau/\tau_0\}$, which may be related to the fact that the accuracy of the activation distance $(x_2 - x_1)$ in this approximation is rather poor, especially when considered over a wide stress range. Cottrell's approximation leads to a reasonable value of $\ln\{\tau/\tau_0\}$, but the temperature dependence of the yield stress appears too weak at relatively low temperatures. The approximation given by the present authors appears to fit the experimental data just as well as either of the other two and the value of $\ln\{\tau/\tau_0\} = 33.5$ appears reasonable. The only very questionable value used in estimating this constant is $n_s = 10^3$. Campbell, Simmons, and Dorn[12] have analyzed the dynamic behavior of a Frank-Read source (ignoring dissipative forces) and predict a nucleation time of about 10^{-9} sec per source. Considering dissipative forces, the physical picture of $n_s = 10^3$ loops/source produced almost instantaneously appears plausible. It is understood, however, that the details of the solution of the dynamic problem concerning the number of loops which can be released instantaneously from a pinned source once set free has yet to be solved. Even if n_s is always 1, the other parameters determining $\ln\{\tau/\tau_0\}$ can be reasonably chosen to predict a value of about 25.

The values of the parameters A, σ_0, and W used to compare theory with experiment show the value of line tension indeed to be about 0.1 of Gb^3 as previously discussed and a maximum atmosphere-dislocation interaction energy of about 0.1 ev. Cottrell[3] assumed $\sigma_0 = 310{,}000$ psi which corresponds to a maximum interaction energy of about 0.15 ev. The slightly lower value found here is to be expected when line energy is also considered in the nucleation problem. It may also represent, however, solute-solute interaction along the continuously pinned dislocation line tending to reduce the maximum interaction energy expected between an isolated interstitial atom and dislocation.

The differences between the present approximation and those due to Cottrell[3] and Haasen[5] have already been enumerated. The model proposed by Fisher[13] considered the effect of line tension in detail upon the breakaway problem. In this model, however, the dislocation is considered to be pinned at each atomic site by impurities with a unique energy of attraction for impurities. The fact that this attraction is a sensitive function of the relative positions of impurity and dislocation as considered in the present study leads to a substantial effect in determining the stress dependence of the activation energy. The present model also considers a simultaneous breakaway of the dislocation over the critical length of dislocation, not a consecutive breakaway process as described by Fisher.[13]

The flattening-out of the temperature dependence of the yield stress below about 100°K as seen in Figs. 6 and 7 represents a distinct deviation from all the approximations considered in this paper. This is probably due to the onset of twinning as the initial mode of deformation in this very low temperature range as discussed by Erickson and Low.[11,14] The flow stress at room temperature and above is probably controlled by stress-induced ordering of solute atoms as discussed by Schoeck and Seeger.[15]

It is interesting to consider the activation volume based upon the model presented here. The activation volume would be given by $(x_2 - x_1)\mathbf{b}l$ where $(x_2 - x_1)$ is a monotonically decreasing function of stress and l either decreases or remains essentially constant with stress depending upon which values of A and W are appropriate. The activation volume, however, enters into all the constants of the polynomials given in Eqs. [12] through [14]. This clearly shows that if one were to consider the strain rate at upper yielding to be given by an equation of the form of Eq. [21], then conventional strain rate change or temperature change type of measurements could hardly yield meaningful results if U were only considered a linear function of σ.

It should be pointed out that Mura and Brittain[16] have concluded that the dislocation line tension and solute atmosphere density along the dislocation are factors also that influence the yielding of ingot iron. Their approach to showing the importance of line tension with respect to upper yielding is considerably different from that taken by the present authors and is thus not discussed in this paper.

In summary, it has been argued that line tension effects should not be neglected in studying the problem of freeing a small segment of dislocation from its impurity atmosphere. A numerical solution to this problem following the original Cottrell-Bilby[1] analysis shows that just as good correlation between experiment and theory can be obtained when line tension is not neglected. The solution found in the present study has been compared directly to the solutions of Cottrell[3] and Haasen[5] which neglect line tension effects. A qualitative discussion concerning the nature of dislocation behavior at the upper yield point has been presented. The parameters used in the discussion of the yield phenomena are consistent with those deduced from the refined treatment of the Cottrell-Bilby[1] analysis.

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